Diagnosing Root Causes and Generating Graphical Explanations by Integrating Temporal Causal Reasoning and CBR

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Abstract. This study proposes a methodology to diagnose the root causes of failures in the domain of oil well drilling. The idea is to combine a Bayesian network, which is generated based on an expert knowledge, with situation-specific knowledge of past failure cases. A causal chain is viewed as a temporal sequence. To test the model's capability, six failure cases from the study's application domain (oil well drilling) are considered and one of them has been picked up as the studying case. The model is applied to diagnose the root causes of the chosen failure case. A temporal reasoning approach has been employed to narrow down the determination of the effective concepts, given the observations. The preliminary results show some advantages of the new model in comparison with the model that integrated a multi relational knowledge model with case based reasoning.

Keywords: Bayesian Network, Case-Based Reasoning, Explanations, Temporal Reasoning

1 Introduction

In complex technical domains with a high level of uncertainty, experts are dealing with types of failures in which implementing ad hoc solutions frequently leads to a reemergence of the problem. Oil well drilling is one such domain. Diagnosing and handling these types of problems requires appropriate focus. A strong interaction is required between the system and an expert who examines system event logs and applies his knowledge of the system to identify the root causes of a given failure [1, 2]. Therefore, in domains such as this, diagnosis of the root causes and possible explanations for these causes is a critical issue.

During the last decades, AI experts have tried to create Machine Learning methods that produce results which are interpretable to the human users [3]. People interpret events in their surrounding environment using a variety of explanations. Therefore, providing diagnostic output in the form of explanations is potentially one of the most important properties of intelligent systems [4].

Case-based reasoning is suitable for capturing and reusing human experiences for complex problem solving, and has earlier shown its success also in the oil drilling domain [5]. However, a pure CBR system suffers from the inability to justify a solution - an explanation that goes beyond referring to the best matching case or cases. Further, CBR represents in itself a knowledge-lean method for case retrieval. A model of general domain knowledge would enable cases to be matched based on semantic rather than purely syntactic criteria. Hence, a general domain model combined with CBR will enable the system to generate targeted explanations for the user as well as for its internal reasoning process. Earlier work in our group has addressed this problem by combining CBR with a semantic network of multi-relational domain knowledge [6]. The created system's architecture, and its implementation in the Java programing language are called Creek and TrollCreek, respectively. A problem with that method was the lack of a formal basis for the semantic network that was used, which made the inference processes within the network difficult to develop and less powerful than wanted. The need for a more formal treatment of uncertainty lead to some initial investigations into how a Bayesian network model could be incorporated [7,8]. The work reported here is the first attempt to seriously develop and test such a combined model. Bayesian network has shown its feasibility to build probabilistic models without introducing unrealistic assumptions of independencies [9]. The probability distribution provided by BN enables the conditioning over any of the variables and supports any direction of reasoning [10]. Also, the Bayesian networks framework includes an inference engine, which, given some evidence, is capable of updating its beliefs [11]. The nature of Bayesian networks allows for some explanations to be given regarding the reasoning process [9]. All this makes BNs a proper candidate for causal reasoning in the diagnosis of root causes [10].

Some researches introduced temporal reasoning into Bayesian networks, which is found useful for diagnosis applications [12]. Temporal reasoning can enhance basic causal reasoning by focusing on the time aspect of diagnosing [13].

In our research we focus on identifying root causes of failures in the domain of oil well drilling. We have considered two approaches for our study, BN-Creek1 and BN-Creek2. BN-Creek1 replaces the multi relational semantic network by a Bayesian network as its knowledge model. The BN-Creek2, which we will focus on in this paper, uses a Bayesian network as a knowledge model in addition to a multi relational semantic network. In BN-Creek2 the general knowledge of causal dependencies is combined with situation-specific knowledge of past failure cases. Then, to make the result more accurate, some features of TrollCreek and Temporal reasoning analysis are employed.

The remainder of the paper is organized as follows: Section 2 outlines related work. Our proposed system is presented in Section 3. Section 4 presents an illustrative example, and section 5 discusses and concludes the paper.

2 Related Work

Been et al., 2014 [3] studied a bridging of the gap between machine learning methods and human's decision-making strategy. They modeled the underlying

data, using a mixture model. They used case based classifiers and BN as two interpretable models to identify the most representative cases and important features. Bruland et al., 2010 [14] studied reasoning under uncertainty in the forms of aleatory and epistemic uncertainty. The aleatory uncertainty works on assigning a probability of a particular state given a known distribution and the epistemic uncertainty refers to cognitive mechanisms of processing knowledge. They advocated the use of Bayesian networks to model aleatory uncertainty and case-based reasoning to handle epistemic uncertainty. They discussed two types of architectures for combining CBR and BN. Houeland et al., [11] focused their research on automatically detecting the robustness and performance of systems which combine case-based reasoning and Bayesian network to solve new problem queries, given the system's current state of uncertainty. They presented an automatic reasoning architecture that uses meta reasoning to achieve their goal. Tran et al., 2008 [15], aimed to assist operators in finding solutions for faults in large-scale communication systems. To determine the cases that share common symptoms, they have used a distributed CBR system.

Aamodt et al., 2014 [16], focused on supporting the processes of retrieval and reuse of past cases, computing similarities, generating indexes, etc. They proposed a BN-powered sub-model as a calculation method that works in parallel with general domain knowledge to satisfy their goal. Petersen et al., 2010 [4] addressed weaknesses of Bayesian network with regard to structural and parametrical changes. They suggested adding case based reasoning functionality to Bayesian networks to better observe changes in behavior. Lacave [9], in his paper, analyzed what has been done to date and what challenges remain to be done in the field of explanation in Bayesian networks. Dørum et al., 2002 [17], focused on prediction problem within oil well drilling to avoid costly failures. They introduced a method for reasoning with time-dependent situations in the form of temporal intervals, within a knowledge intensive CBR framework. Their system gives warnings to the user when an unwanted event may be approaching.

Casey is another system that combines CBR with a general domain knowledge model, particularly a causal model. Hence, for the situations that CBR is not able to retrieve a qualified case, CASEY uses the causal model as a second attempt to solve the problem [18]. Long [13], utilized temporal reasoning in Heart Disease Program and discussed about the domain's existing issues and solutions in integrating temporal reasoning with pseudo Bayesian probabilistic reasoning.

Some of the aforementioned researches employ BN in different segments of CBR. Our research, as the others, takes advantage of both BN and CBR's features but in a way that BN is used as a causal relation knowledge model associated with cases. The temporal reasoning is employed to increase the accuracy of the root causes diagnosing process.

3 Proposed Model Architecture

The final goals of this study are prediction of failure root causes and to generate explanations given the observed symptoms or errors.

The main structure of a Bayesian network has been designed in order to express the elements' relations and calculate the updated beliefs based on the prior probabilities assigned by a field expert. The domain concepts are presented by nodes and their causal relations are shown by arrows. A parent causes a child, and each node represents the current belief of the network given its parents. Our approach in the first place, views the BN as a different type of, and a replacement for the knowledge model in TrollCreek (BN-Creek). Then, integrates TrollCreek case retrieval results with the BN-Creek results to get benefit from the other type of relations that been considered in TrollCreek. Fig. 1 depicts the graphical structure for the proposed approach. The filled and not filled circles are indicators of Bayesian network nodes and cases, respectively. TrollCreek and the present approach are extracting the cases from the raw data, but the main difference between them is their knowledge models. TrollCreek uses a multi relational semantic based knowledge model while the new approach uses a probabilistic causal model as its knowledge model.



Fig. 1. The BNCreek knowledge structure

The field expert's knowledge has been exploited to create the aforementioned BN. The causal relations between the oil well drilling process' concepts were identified by the expert and were used as the prior probabilities of the BN.

The main task in this study is to answer the query of: "What is the whole probability distribution over variable X given evidence e?". In other words, the most plausible causes of the failure under study, given some observations, is desired. In our approach, the mentioned query will be answered in the following three steps.

- Step one: This step utilizes BN to calculate a temporal probability distribution of the new case.
 - By creating a new case, a copy of the domain's prior Bayesian network is assigned to it. The inference process is started, by applying each of the

observed concepts as evidence in the network. Then the network's beliefs will be updated given those evidence and the result will be shown as the posterior distribution (PD) of the network, corresponding to that specific case.

- Step two: In this step, we utilize the CBR's capability of employing the past experiences, aimed to improve the BN's accuracy in suggesting the root causes.

TrollCreek is used to retrieve the most similar cases to the new one. The bestmatched case is considered and an impact factor is assigned to its recorded PD, based on its similarity degree.

TrollCreek uses MAC-FAC method [18] to retrieve the cases. As the MAC phase each of the findings from the testing case are compared to all the findings from the retrieved case aimed to find similar findings as many as possible. The Eq.1 illustrates the similarity assessment formula:

$$sim(C_{IN}, C_{RE}) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} sim(f_i, f_j) * relevance factor_{f_j}}{\sum_{i=1}^{m} relevance factor_{f_j}}$$
(1)

In Eq.1 the C_{IN} stands for the under study case and and C_{RE} demonstrates the retrieved case. n and f_i , m and f_j are the number of findings and the finding's number in the C_{IN} and C_{RE} , respectively. The $sim(C_{IN}, C_{RE})$ is equal to 1 if $f_i = f_j$, otherwise it's value would be 0. The relevance factor is a number that combines the predictive strength and importance of a feature for a stored case and comes from the expert [6].

The FAC phase considers the paths in the semantic network that represent relation sequences between un-identical features. Based on a method for calculating the closeness between two features at each end of such a sequence, the two features are given a local similarity score.

- Step three: This step integrates the probability distributions from the first two steps and calculates the new case's finalized probability distribution. In other words, in this step we have added the CBR's capability in employing the past experiences, to improve the BN's accuracy in suggesting the root causes. The result of step three is the system's outcome.

Eq.2 integrates the effect of the pure BN and CBR from the first two steps and generates the finalized posterior distribution for the new case.

$$PP_{jf} = \frac{\sum_{i=1}^{k} PP_{ji} * \alpha_i}{\sum_{i=1}^{k} \alpha_i}$$
(2)

In Eq.2 the PP stands for the posterior probabilities which are the elements of the Posterior distribution. The $0 < \alpha < 1$ is the impact factor that is larger for the cases with higher similarity. The 'k' is the number of PDs that are integrated together and would be higher than two in a situation that the expert wants to involve the effect of less matched cases. 'j' and 'i' a're the indicators of a specific PP in a PD and the PD's number, respectively. Consequently, the PP_{jf} stands for the finalized posterior probability of the PP number 'j'. The index 'f' stands for finalized PP.

After completion of the third step, the finalized updated network's beliefs (PD) are achieved. Using the final PD, the strengths of the potential root causes are listed and are given to the expert for assessment.

Before suggesting the list of candidate concepts to the expert, a narrowing down process can do on the list, to improve accuracy, by applying temporal analysis.

Temporal probabilistic reasoning analysis considers the aspect of time for events. One of the main effects of this perspective is the consideration of older observations as extraneous observations. Depending on the dynamic nature of the system and considering a long enough time between the observation time and the time point of interest, such observations can be ignored without any loss in the accuracy of the conclusion [12].

In this work the list of concepts from the three aforementioned steps are considered and the time sequence of their concepts are extracted from the raw data. Then, the concepts that passed the expert threshold are removed from the list. The next chapter illustrates how temporal reasoning is incorporated in this research.

Based on the selected root causes, the explanation path between the given evidence and the considered root cause will be presented.



Fig. 2. Some instantiated parts of BNCreek

4 An illustrating example

This section provides an example to illustrate how BNCreek works. To test the proposed method's capability we exploited 6 drilling failure cases and considered one of them as the studying case, namely *case wellbore clean02*.

The Bayesian network of the drilling process, which is created based on a field expert's knowledge is employed. Five different types of concepts are used in the network. Symptom(s), Static Symptom(ss), Error(err) and Failure(f) which are observable concepts and cannot be the root causes of a situation as they express situations. The fifth category of concepts, internal parameters(i), are chosen to be the potential root causes of the failure, as they represent unobservable parameters. The internal parameters have been identified by the drilling experts, for some other purposes but introducing the root causes. Therefore, not all of them can be the root causes of a failure, necessarily. On the other hand, they are the most proper candidates to satisfy the concept of root cause. Then considering the main focus of the study, which is illustrating the capability of BN to serve as a replacement for the multi-relational knowledge model, we have accepted the internal parameters as the candidates of root causes in this study. Additional efforts are being conducted to identify new type of concepts, which will serve as stronger candidates for root causes.

Employing BNCreek: By employing BNCreek to diagnose the root causes of the failure *case wellbore clean02*, the 3 aforementioned steps are implemented as follows:

Step One: First of all, one copy of the drilling process' Bayesian network is prepared for the under-studying case, called the *case wellbore clean02 BN*. This copy demonstrates the prior beliefs of the network based on the experts beliefs. Besides, the observed Mechanical Restrictions of the *case wellbore clean02* are retrieved from the case description.

Fig.2 demonstrates a small part of the case wellbore clean02 BN after updating the networks beliefs, given the above-mentioned evidence. The networks updated beliefs are called the temporal posterior distribution of the case wellbore clean02. The figure illustrates the observations of the present case which connect the cases to the Bayesian network. Let us pick the internal parameter Wellbore Enlarged (i) as a sample to illustrate the network's Bayesian analysis to update the beliefs in BNCreek model. The goal is to figure out the P(Wellbore Enlarged (i) | evidence). To reach our goal we need to consider all concepts that are related to and affected on the Wellbore Enlarged (i) and calculate the posterior probability of the considered concept by inference into the network.

As it is observable from Fig.2, the Wellbore Enlarged (i) is caused by Fm Soft(s), Mud Water Activity High(ss) and Wellbore Wall Erosion(i), then it is dependent on them. Moreover, each of these concepts is dependent on some other concepts, e.g. the Wellbore Wall Erosion(i) is dependent on Side Force High(s) and Build/Drop Section Inside Openhole(ss) and this sequence continues. On the other hand, there are nine evidence that affect the beliefs of the network. Consequently, considering the number of involved concepts and the enlargement of the network, manual computation of the P(Wellbore Enlarged (i) | evidence) is

almost impossible. Therefore, we have utilized an automatic calculation employing Genie and Smile tool. Finally, considering all the dependent concepts, their prior probabilities and the nine evidence the result of doing an exact inference in the network illustrates the value of %51 as the temporal posterior probability of the *Wellbore Enlarged* (i). The temporal posterior distribution for the rest of the internal parameters are listed in the second column of Table 1.

Step Two: The similarity matching section of the CBR is employed here. The case wellbore clean02 is tested with the CBR to find the most similar case from the case base. By applying the method, the case wellbore clean01 is suggested as the best match case. The total similarity matching degree between aforementioned cases has been calculated employing Eq.1 and the FAC phase. Finally, the 23% of similarity is achieved. As the final part of step two, the case wellbore clean01 recorded PD has been retrieved and its internal parameters have been listed to be used in the next step, see the third column of Table 1.

Table1: list of possible root causes			
Root Causes	WelboreClean02	WelboreClean01	WelboreClean02
			Final
Activity Of Tripping (i)	100 %	94%	61%
Activity Of Reaming (i)	85%	85%	52%
Mechanical Restriction (i)	83%	78%	50%
BHA Balled(i)	98%	12 %	50%
Time Long (i)	70%	40%	40%
Sliding Mode (i)	66 %	50%	39%
Wellbore Wall Erosion (i)	56%	54%	34%
Drill String Cyclic Load High(i)	54%	54%	33%
Wellbore Enlarged(i)	51 %	47%	30%
Cyclic Load High (i)	50%	50%	31%
Cuttings Concentration Low (i)	48%	46 %	29%
Shale Swelling Invisible (i)	50%	37%	29%
ECD Surge High (i)	43%	43%	26%
Fm Boundary (i)	30%	99%	26%
Accumulated Barite (i)	42%	35%	25%
Wellbore Ledge/Shoulder (i)	40%	40%	25%
Cuttings Concentration high (i)	42%	31%	25%
Well Complex (i)	40%	33%	23%
Cavings Produced (i)	39%	19%	22%
Mud LGSC High (i)	38 %	21%	21%
Shale Brittle(i)	32 %	32%	20%
Cuttings Bed Compact (i)	32%	25%	19%
Fm Gas Bearing Zone (i)	30 %	30%	18%
Cavings Blocky (i)	25%	43%	17%
Cement Sheath Quality Low (i)	28%	28%	17%
Mud Gas Content High (i)	28%	28%	17%
Bending Of BHA (i)	22%	22%	14%
Cavings On Shaker (i)	24%	12%	13%
Filter Cake Thick (i)	20 %	21%	12%
Cement Insufficiently Displaced (i)	20%	20%	12%
Cuttings Bed Erosion Low (i)	20%	15%	12%
Fm Fault Intersected (i)	18%	22%	11%
WellboreWall Restricted(i)	16%	27%	11%
Motor Erosion (i)	17 %	19%	10%
Cuttings On Shaker (i)	17 %	13%	10%
Csg Ann P High (i)	16%	14 %	10%
Fm Above Charged(i)	12 %	11 %	7%
Cuttings Bed Erosion High (i)	10%	2%	5%
Bit Vibration (i)	0%	16 %	2%
Wellbore Restricted(i)	0 %	9%	1%

Step Three: The potential root causes of the case wellbore clean 02 and case wellbore clean 01 are considered and the impact factor 1 and 0.23 are assigned to them respectively. To illustrate more details, let us continue by the Wellbore Enlarged (i) with the temporal PD value of 51% which was obtained from step one. The recorded PD of the matched case shows 47% of possibility for wellbore

Enlaged obtained from step two. Considering the similarity degree between the two cases, the impact factor 23% has been assigned to the retrieved case and 100% to the inputed one. Consequently, by applying Eq.2, the final posterior distribution of the *wellbore Enlarged(i)* has been achieved 30%. *case wellbore clean02* is generated and recorded as a part of case description. Table 1, column 4 demonstrates a list of the drilling domain's final PD for *case wellbore clean02*. The expert would assess this list.



Fig. 3. Sample paths between two evidence and a root cause candidate in BNCreek

Employing temporal reasoning: Fig.3 illustrates some elements of downhole drilling problems as a sequence of concepts which is a part of the drilling failure's Bayesian network. Following example shows how temporal reasoning increases the accuracy of diagnosing the root causes of drilling failures.

Adding a time sequence to the causal relations will provide a threshold for the impact level of the observation. Table 2 illustrates the hypothetical time sequence which is considered in this study.

Table2: time sequence			
Side Force High(s)	01:00 am		
Wellbore Wall Erosion(i)	01:30 am		
Wellbore Enlarged(i)	01:40 am		
Accumulated Cuttings(err)	02:00 am		
Mechanical Restriction(i)	02:15 am		
Took Weight(s)	02:30 am		

Due to the nature of the problem, assume that passing one hour is an enough time to ignore the older observations without loss in the accuracy of the conclusion. Then, for diagnosing the root cause of the 'Took Weight (s)' the 'Side Force High(s)' could be removed from the root causes candidates.

To use the temporal reasoning results we need to know the time sequence of observations and the threshold of impact level of the observations. In the explained example the considered time for the observations and the threshold for impact level are hypothetical. The real time sequence of the cause-effects should be obtained from the raw data during the failure case extraction process. Moreover, the threshold of the impact level would be determined by the expert. Based on the raw data, the expert should decide how long it takes till the effect of an observation becomes small enough to be negligible.

5 Discussion and Conclusion

Two main advantages and one of the weaknesses of BNCreek are being addressed. The first and the most important advantage of BNCreek in comparison with TrollCreek is its global view to the network's beliefs. BNCreek takes benefits from the dynamic information flow between the concepts as it has employed the Bayesian network. It means that the new observed information will affect the beliefs of all the related concepts in the network which in this study leads to the global and dynamic adjustment of the potential root causes given the observations. While TrollCreek method has a local perspective. It means it uses the static strength of the relations. The second advantage of developed system is the ability of considering the effect of more than one evidence in the network at the same time which leads to simulating the logical operators, i.e. "And", "Or" which is an important capability due to our studying domain's demands. While importing such operators to Trollcreek's knowledge model needs a significant changes at the model. On the other hand, the knowledge model that is employed at TrollCreek covers other types of relations in addition to the causal ones, which results in covering more important details in addition to the cause and effects.

As a conclusion BNCreek showed a more flexible and stronger capability to make inferences and to make a diagnosis of the root causes in comparison with TrollCreek method.

Our future study will focus on more integrating temporal reasoning with inference in the BN and make an effort to include other types of relations, a combination of the BN and knowledge model.

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